Some Important Mathematicians

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A Galaxy of Stars

- Euclid (fl. 300 B.C.).
- Archimedes (287 BC–212 BC).
- Pierre de Fermat (1601– 1665).
- Isaac Newton (1643–1727).
- Leonhard Euler (1707–1783).
- Johann Carl Friedrich Gauss (1777–1855).
- Évariste Galois (1811–1832).
- Niels Henrik Abel (1802–1829).
- David Hilbert (1862–1943).
- Srinivasa Iyengar Ramanujan (1887–1920).
- Andrey Nikolaevich Kolmogorov (1903–1987).
- John von Neumann (1903–1957).

The discussions will be based on Wikipedia.



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Euclid (fl. 300 BC)

"There is no Royal Road to geometry".

- Father of Geometry: (but, not the "originator".)
- Elements: a paradigm.
  - One of the most influential works in the history of mathematics.
  - Built the edifice of geometry from five axioms.
    - Later, variants of the fifth axiom gave rise to alternative geometries.
  - Gave rise to the notion of rigorous mathematical proofs.
    - Technique of reductio ad absurdum.
    - Deductive reasoning.
  - Contains important results in number theory.
    - Primes are infinite; algorithm for finding gcd.
    - Euclid's lemma: if $p|ab$, then $p|a$ or $p|b$.
- Credited with other works some of which (such as a book on conic sections) have been lost.



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Archimedes (287 BC–212 BC)

"Give me a place to stand on, and I will move the Earth."

- Leading mathematician of (western) antiquity; also, physicist, engineer and astronomer.
- Mathematics.
  - Method of exhaustion.
    - Formula for the area of a circle; approximation of π. Consideration of inscribed and circumscribed polygons.
    - Other applications.
  - Formulae for the volumes of surfaces of revolution.
  - Archimedean property of real numbers: any magnitude when added to itself enough times will exceed any given magnitude.
- Physics.
  - Hydrostatics: law of floatation.
  - Principle of levers (but, not the "inventor" of levers).



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Pierre de Fermat (1601– 1665)

French lawyer and an ‘amateur’ mathematician.

- Considered as co-founder (with Blaise Pascal) of probability theory.
  - First to perform a rigorous probability calculation: Pr[one six in 4 throws of a single die] $>$ Pr[double six in 24 throws of double dice].
- Analytic geometry: predates Descartes but, recognised posthumously.
- Developed a method for determining maxima, minima, and tangents to various curves.
- Obtained a technique for finding the centres of gravity of various plane and solid figures.



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Contributions to number theory.

- Fermat's little theorem: $a^p \equiv a \bmod p$.
- Method of infinite descent. Based on showing that if $P(a)$ does not hold, then there is a $b < a$, such that $P(b)$ does not hold.
  - Used to show that $x^4 + y^4 = z^4$ does not have a solution in integers x, y and z.
  - Two-square theorem: Any odd prime p is expressible as the sum of two squares iff $p \equiv 1 \bmod 4$.
- Fermat's last theorem. For positive integer $n \geq 3$, there is no solution in integers x, y and z to the equation $x^n + y^n = z^n$.

"I have a truly marvelous demonstration of this proposition which this margin is too small to contain."



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Isaac Newton (1643 – 1727)

Mathematician, physicist and astronomer.

- Mathematical Principles of Natural Philosophy (Principia): a paradigm.
  - Laws of motion and the law of universal gravitation.
  - Motions of objects on Earth and of celestial bodies are governed by the same laws.
- Decomposition of white light.
- Mathematics.
  - Development of differential and integral calculus. (Credit is shared by Gottfried Leibniz.)
  - Generalised binomial theorem.
  - Approximating the roots of a function.
  - Contributed to the study of power series.



Isaac Newton (1643 – 1727)

"If I have seen further it is only by standing on the shoulders of giants."

"I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."



Leonhard Euler (1707–1783)

"He calculated without any apparent effort, just as men breathe, as eagles sustain themselves in the air."

– François Arago on Euler.

Prolific contributions to different subjects.

- Geometry, infinitesimal calculus, trigonometry, algebra, number theory, graph theory and other areas.
- Mechanics, fluid dynamics, optics, and astronomy.



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- Mathematical notation:
  - Notion of a function and the notation $f(x)$; use of e to denote base of natural logarithms; modern notation for trigonometric functions; $\sum$ for summation; and i for the imaginary unit.
- Influential work on power series; expression of functions (such as e^x) as sums of infinitely many terms.
- Introduced gamma functions; invented calculus of variations; ...
- Complex analysis: exponential function for complex numbers.
  $e^{ix} = \cos x + i \sin x$;
  special case (Euler's identity): $e^{i\pi} + 1 = 0$
  ("the most remarkable formula in mathematics" – Richard Feynman).



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- Pioneered analytic number theory.
  - Hypergeometric series; the analytic theory of continued fractions, ...
  - Sum of reciprocals of primes diverges.
- Number theory.
  - Proved Fermat's little theorem; Fermat's theorem on sums of two squares; ...
  - Introduced the totient function $\phi(n)$.
  - Conjectured the law of quadratic reciprocity.
- Graph theory and combinatorics.
  - The first theorem of graph theory: solution to the problem of the seven bridges of Königsberg.
  - Euler's formula for planar graphs: $V - E + F = 2$.
  - Euler's conjecture: For $n \geq 6$, there does not exist any pair of $n \times n$ orthogonal latin squares.



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Johann Carl Friedrich Gauss (1777–1855)

"*Few, but ripe.*"

The prince of mathematicians and "the queen of sciences."

- Disquisitiones Arithmeticae: a paradigm.
  - Fundamental in consolidating number theory as a discipline.
  - Had a long-term influence in shaping the field.
  - Introduced modular arithmetic and the notation $\equiv$.
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- Geometry.
  - Showed that heptadecagon (regular polygon of 17 sides) can be constructed using straight-edge and compass.
  - Non-Euclidean geometry.
- Statistics and linear algebra.
  - Ideas introduced in the “Theory of motion of the celestial bodies moving in conic sections around the sun”.
  - Method of least squares; method of maximum likelihood; and the normal distribution; method of elimination to solve a system of linear equations.
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- Methods for astronomical calculations.
- Differential geometry.



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Évariste Galois (1811–1832)

"Don't cry, Alfred! I need all my courage to die at twenty."

"This letter, if judged by the novelty and profundity of ideas it contains, is perhaps the most substantial piece of writing in the whole literature of mankind."

– Hermann Weyl about the last letter written by Galois.

- Group theory: idea of normal subgroup, left and right cosets.
- Galois theory.
  - Relation between the algebraic solution to a polynomial equation and a group of permutations associated with the roots of the polynomial.
- A necessary and sufficient condition for a polynomial to be solvable by radicals.
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"By studying the masters, not their pupils."

"He is like the fox, who effaces his tracks in the sand with his tail."

– Abel said famously of Gauss's writing style.

- Invented group theory (independent of Galois).
- Proved the impossibility of solving the quintic (or higher degree) equation in radicals.
  - Solved a long standing open problem.
- Proof of the binomial theorem valid for all numbers.
- Important work on elliptic functions.



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David Hilbert (1862–1943)

"We must know – we will know!"

- One of the founders of proof theory and mathematical logic.
- Hilbert basis theorem: every ideal in a ring of multi-variate polynomials over a Noetherian ring is finitely generated.
  - A proof of existence without an explicit construction.
- Hilbert's Nullstellensatz.
  - The fundamental connection between algebra and geometry.
  - Generalization of the fundamental theorem of algebra.
- Unified the field of algebraic number theory.
- Solved Waring's problem: For every natural number k, there is a positive integer s such that every natural number can be written as the sum of at most s k^{th} powers of natural numbers.
  - Every natural number can be written as sum of at most 4 squares, 9 cubes, or 19 fourth powers, ...



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- Solved Waring's problem: For every natural number k, there is a positive integer s such that every natural number can be written as the sum of at most s k^{th} powers of natural numbers.
  - Every natural number can be written as sum of at most 4 squares, 9 cubes, or 19 fourth powers, ...



David Hilbert (1862–1943)

- Hilbert's problems.
  - A list of 23 unsolved problems some of which were presented at the ICM in Paris in 1900.
- Hilbert's program: to show that all of mathematics can be deductively obtained from a provably consistent finite set of axioms.
- Functional analysis: follows from his studies of differential and integral equations.
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- Mathematical physics.
  - Axiomatic derivations of the field equations of gravity.
  - Anticipated and assisted advances in quantum mechanics.
- Class field theory.



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"An equation for me has no meaning unless it expresses a thought of God."

"Every positive integer is one of Ramanujan's personal friends."

– John Littlewood.

- Contributions to number theory, infinite series and continued fractions.
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  - A small number were actually false and some were already known.
  - Most of his claims have now been proven correct.
  - Applications to crystallography and string theory.



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- Ramanujan's four notebooks.
  - First notebook: 351 pages, 16 chapters and some unorganized material.
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  - Ramanujan was capable of proving most of his results, but, chose not to do so.
- Unusual formulas are a characteristic feature of his work, e.g.,

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103 + 26390k)}{(k!)^4 396^{4k}}.$$

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- 'Foundations of the Theory of Probability.'
  - Laid the modern axiomatic foundations of probability theory.
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- Linear programming: developed the theory of duality.
- Mathematical statistics.
- Numerical hydrodynamics.
- Contributed to the development of the Monte Carlo method.
  - Proposed early constructions of pseudo-random numbers.
- Manhattan project, thermo-nuclear reaction, fission bomb (Fuchs-von Neumann patent).

